**OPER 510**

**Introduction to Math Programming**

**Final Report**

**Project Group Seven (AFIT Team Sevenworks)**

Capt Marc Chalé

CPT Courtney Reynolds

1st Lt Ryan Bruns

1st Lt Kevin Jagoda

1st Lt Stephen Lee

(A Highlighted section denotes something that requires additional reference or attention. It can be addressed later so I don’t get out of the writing groove trying to find the figure

Contents

[Executive Summary 4](#_Toc531190626)

[Methodology & Overall Approach 5](#_Toc531190627)

[Team Roles Description 10](#_Toc531190628)

[Base Case Results 11](#_Toc531190629)

[Interpretation 11](#_Toc531190630)

[Constraints, Limitations, Assumptions 11](#_Toc531190631)

[Results 11](#_Toc531190632)

[Sensitivity Analysis 11](#_Toc531190633)

[Major Conclusions 11](#_Toc531190634)

[What-If #1 Case Results 12](#_Toc531190635)

[Interpretation 12](#_Toc531190636)

[Constraints, Limitations, Assumptions 12](#_Toc531190637)

[Results 12](#_Toc531190638)

[Sensitivity Analysis 12](#_Toc531190639)

[Major Conclusions 12](#_Toc531190640)

[What If #2 Case Results 13](#_Toc531190641)

[Interpretation 13](#_Toc531190642)

[Constraints, Limitations, Assumptions 13](#_Toc531190643)

[Results 13](#_Toc531190644)

[Sensitivity Analysis 13](#_Toc531190645)

[Major Conclusions 13](#_Toc531190646)

[What If #3 Case Results 14](#_Toc531190647)

[Interpretation 14](#_Toc531190648)

[Constraints, Limitations, Assumptions 14](#_Toc531190649)

[Results 14](#_Toc531190650)

[Sensitivity Analysis 14](#_Toc531190651)

[Major Conclusions 14](#_Toc531190652)

[Conclusions 15](#_Toc531190653)

# Executive Summary

# 

The CEO of the MCS Corporations contracted a group of management consultants, AFIT Sevenworks, to examine all facets of its production for its DinoBall toy. MCS is currently able to facilitate 8 retail stores and 3 production facilities for selling this product. The company intends to meet all demand requirements for the DinoBall. Furthermore, and before continuing its current production plans, the corporate officers of MCS realized there are variable production parameters to include (volume, number facilities, shipping routing of product, wear and tear on lines, production infrastructure upgrades) that it wants to examine, where, if changed, a reduction in objective of lowest production and shipping cost of DinoBall is possible. Additionally, MCS has adopted a green initiative and has the option to stand up a new production technology, FastProd, where if implemented, has the ability to lower per unit production costs and increase production rates. The company contracted with a management consulting time at AFIT known as Sevenworks to consider and report on the following:

The production parameters, or decision variables, that are subject to change pending a comprehensive analysis by Sevenworks are: the quantity of DinoBalls shipped from each existing production facility to each retail store, the quantity of trucks used for each route in each month (12 month period), and the decision to implement the FastProd technology at a given production facility at some time.

In a review and comprehensive modeling analysis of the production parameters of MCS for DinoBall, we will consider 4 scenarios: a base (existing case), as well as three additional what-if cases.

We will formulate this type problem with these parameters (decision variables)

BASE CASE: AFIT Sevenworks considered a baseline and three what if cases scenarios. Each facility produced 33.3 percent of the output. For the base case, Sevenworks, formulates a (insert type of model here, transport, integer, transship) between the current state of all production facilities, retail stores, and truck logistics as is. Additionally, as part of this case, Sevenworks considered the corporate policy requirement that each of the three production facilities produced equally one third of the total volume of DinoBalls.

ALTERNATVE CASE 1: Sevenworks examined the first alternative case on the effect of eliminating the corporate policy of producing exactly one third of the total DinoBall production at each facility affects total production cost and output rate with FastProd. (Try and insert some sort of result number/figure here).

ALTERNATIVE CASE 2: Sevenworks studied the second alternative case of implementing FastProd technology of any combination of each of the production lines 1, 2, 3 where MCS sells any of existing assembly lines to implement FastProd. (Try and insert some sort of result number/figure here).

ALTERNATIVE CASE 3: Sevenworks investigated the third alternative case of a goal program tradeoff analysis where there are competing objectives of minimizing production cost with minimizing production emissions. (Try and insert some sort of result number/figure here).

BLUF: AFIT SevenWorks considered all these cases and provides results, conclusions and advisory recommendations in the following sections of this report to provide MCS corporate

(The bluf needs to have recommendations final dumped here.)

# Methodology & Overall Approach

The Sevenworks team first established specific roles that aligned with subject matter in specific areas that aided the completion of this study. It ensured accountability and a foundation for analysis excellence early on:

## Sevenworks Team Roles and Description

* Project Manager Marc Chalé
  + Scheduling
  + Scoping the work
  + Tasking
  + Liaison to AFIT Sevenworks Executives
* Chief Analyst Courtney Reynolds:
  + Directing technical work
  + Technical advising
  + Logistics SME
  + Communicating findings
  + Analysis team decision authority
* Business Analyst Ryan Bruns
  + Research constraints, limitations, assumptions
  + Interpret problem for OR Analysts
  + Provide feedback on relevance of work
  + Draw conclusions from results/analysis
* OR Analyst Stephen Lee
  + Interpret problem
  + Formulate mathematical model
  + Implement excel model
  + Provide technical expertise in Excel
* OR Analyst Daniel Jagoda
  + Interpret problem
  + Formulate mathematical model
  + Implement Lingo model
  + Provide technical expertise in Lingo

While these roles established overall areas of responsibility for each person, all of the Sevenworks team worked to help other team members as needed in different role areas to complete the analysis study of the DinoBall supply chain on time.

## Historical Data

The Sevenworks team then examined the historical data on supply chain behavior provided by MCS . This included a data review of each item listed below. Sevenworks input this data into tabular form in order to use as inputs for a model that required inputs of that information.

|  |
| --- |
| * Monthly demand at each retail store |
| * Fuel cost from production |
| * Existing production line specifications |
| * + DinoBall production rate |
| * + Hourly energy use |
| * + Production cost per DinoBall |
| * + Emissions units per DinoBall |
| * Shipping distances from production facility to retail store |
| * Fuel requirement to ship from production facility to retail store |
| * FastProd line specifications |
| * + DinoBall Production rate |
| * + Hourly energy use |
| * + Production cost per DinoBall |
| * + Emissions units per DinoBall |

## Constraints

The Sevenworks team derived constraints through analysis of historical data, and problem statement interpretation. Each constraint is listed with superscript b, 1, 2, 3, respectively, to annotate which scenario the constraint applies to; base case, and what if scenario 1, 2, 3. The constraints include:

|  |
| --- |
| 1. Aggregated monthly production limit at each facilityb123 |
| 1. Truck capacity of 250 DinoBalls per Trip b123 |
| 1. Equal production at each production facility b23 |

## Limitations

The Sevenworks team identified **limitations** of modeling the supply chain for DinoBall with an MS Excel mixed integer problem Each limitation is listed with superscript b, 1, 2, 3, respectively, to annotate which scenario the limitation applies to; base case, and what if scenario 1, 2, 3. The constraints include:

|  |
| --- |
| 1. Infeasible to force integer quantities of DinoBalls shipped from production facility to retail store b23    1. This assumption, if enforced, would directly conflict with constraint 3 |
| 1. Floating point error in MS Excel calculations |
| 1. Generalized Reduced Gradient Non-Linear algorithm uses convergence criteria of 10-4 b123 |

Evidently, the optimal solution of volume DinoBalls shipped from production facilities to retail stores, while meeting store demands, in this model may be a non-integer quantity. Sevenworks concluded that this is an acceptable and necessary limitation to the model that does not significantly reduce the fidelity or accuracy of the results.

## Assumptions

The Sevenworks team developed model assumptions in order to properly solve the problem. These assumptions are well reasoned and logically defensible statements regarding the true behavior of the system under investigation. Each assumption is listed with superscript b, 1, 2, 3, respectively, to annotate which scenario the assumption applies to; base case, and what if scenario 1, 2, 3. The constraints include:

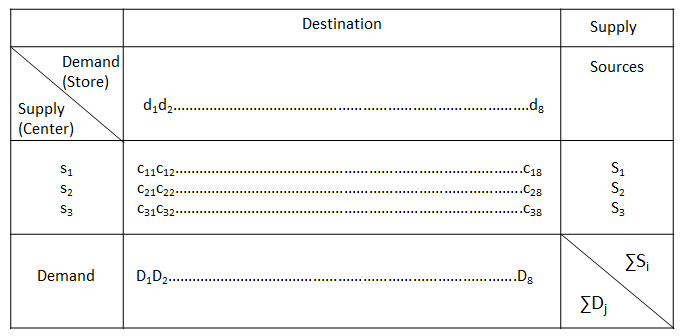
|  |
| --- |
| * Historical data (see section above) is a reliable prediction of future system behavior |
| * Planned production volumes are met each month; there is no assembly line down time b123 * Required transportation is met by LM Trucking, and there are no delays to transportation b123 |

## Model Synthesis

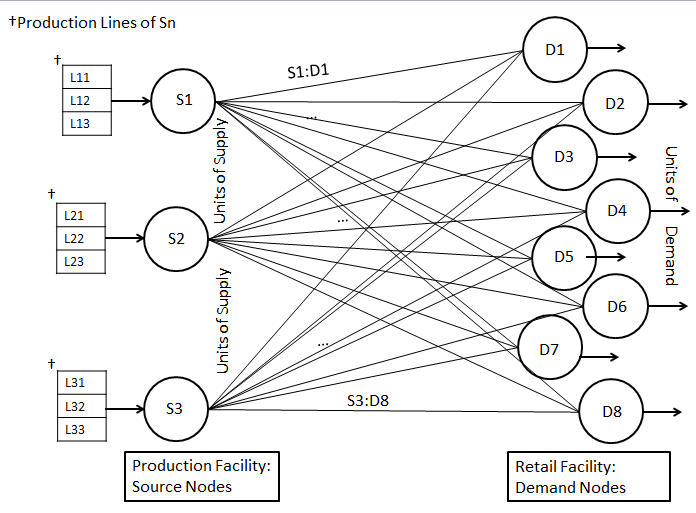
Sevenworks compiled the constraints, limitations, assumptions, and problem background data and then searched for a model that would suffice for incorporating the entirety of this information to produce a logical, accurate, complete results for each case where MCS requested a solution: the base (as-is) case, and the three “What-If” cases.

One method in operations research linear programming that is widely used to model the flow of units of product, in the case of MCS DinoBalls, between production centers, or supply centers, and stores, or demand centers, is the **transportation model**. This model format focuses on assigning specific quantities of some arbitrary unit (in the case of MCS DinoBall) originating at a supply node to a demand node at a some given cost to transport that can very between those nodes. The **objective** of the model is to minimize or maximize some sort of cost or revenue associated with shipping each of the units from Di to Sj and that when summed over all units shipped, maximizes or minimizes the total cost or revenue associated with the transportation of all units. In the case of the DinoBall, Sevenworks determined the **objective** was to minimize shipping cost across all units going from the three supply production sites to the eight demand or store sites.

In the case of MCS and the DinoBall, Sevenworks established the supply nodes Sn to be the production centers in Troy, Newark, and Harrisburg, and the demand nodes Dn the eight cities where stores of DinoBall exist (Pittsburgh, Et al.). Some quantities *s1, s2, s3* are available at each supply source, these factories (center) supply certain the S product. The problem requires transporting the products to demand centers (destinations*) d1, d2, d3* with the number of requested units D1,D2,D3. The cost of transporting one unit of project from each supply *Si* to satisfy demand *dk* is *Cij* shown below.

  
**Figure 2: Destination (Store), Demand Centers and Supplier (Production Facility) Required Qtys**

As shown in Figure 1, there arcs to each demand node stem from each supply node representing the possibility of shipping from a given supply node to a demand node. The arcs or lines that span the gap between the supply and demand node each have a unique associated cost of shipping the unit (Put this in appendix).



**Figure 1: A Network Diagram of this transportation problem**

Sevenworks determined DinoBalls have a requirement to be shipped by truck which costs money, so use of a truck that can ship up to 250 units incurs a per truck cost. If less than the cap of the truck is shipped from some supply to demand node, the entire truck is still paid for. If more than 250 units are shipped from some supply to demand node, the number of trucks used and cost incurred must be number units divided by 250 and rounded up to next whole number. The cost table per truck used going from Si to Dj is listed in the appendix.

Sevenworks then formulated the transportation problem for the baseline case, modeling the production parameters as is.

Decision Variables

|  |
| --- |
| : DinoBalls shipped from production facility Si to store Dj on month k  : Truck sent from production facility Si to store Dj on month k. |

Constants:

|  |  |
| --- | --- |
| : unit shipping cost per DinoBall from Si to Dj | : demand of DinoBall at a store j, month k. |
| : fuel cost from Si to Dj per truck used | : the unit energy cost for factory i |

Objective Function: Minimize z: the year’s, all twelve months, total cost of the supply chain in dollars

|  |
| --- |
| Min *z*1= Unit Production Costs (energy usage) + Shipping Costs (from *Si* to *Dj*, trucks and weight) |
|  |

Subject to the Constraints:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Production Limits |  | |  | |
|  |  | |  | |
|  | | | | |
| Demand |  | |  | |
| (Straight demand\*) all demand must be satisfied. | | | | |
| (each factory each shares 1/3 demand), | | | | |
|  | | | | |
| One Truck Required Per 250 Units Shipped | |  | |  |
|  | |  | |  |
|  | |  | |  |
| Equality of Supply | | Equality of Demand | |  |
|  | |  | |  |
|  | |  | |  |
| Trucks Used is an Integer | | Non-Negativity Trucks Used | | Non-Negativity, Units Shipped |
|  | |  | |  |
|  | |  | |  |

You can make the case to move this over case two, group discussion.

**Objective 2: The Reduce Emissions Case**

|  |  |  |
| --- | --- | --- |
| Production Limits |  |  |
|  | | |
| Where is the pollution from factory *i* with or without FastProd | | |

Also, before production, there is an associated production cost per unit that must be considered and varies between each factory depending on factors such as output rate, energy use per hour (Put this in appendix).

Now that Sevenworks formulated the problem, we ran the model for the base case, and subsequent what if cases using Microsoft Excel. Sevenworks will explain the results of this model in the following case sections: the baseline, as well as “What-If” 1,2 and 3.

Emissions are considered by Sevenworks as MCS wants to think long term about going green but not in the baseline case.

We can use a Microsoft excel solver add in or modeling optimization software such as lingo to aid this process. Explain now that we have the base case we can tweak the results from the base case to model each “what if” case 1,2,3 as MCS priorities change.

# Base Case Results

The base case for this investigation asks *what is the least expensive way to manufacture DinoBalls at each production facility, and transport them to retail stores, such that all customer demands are met?* The base case specifically requires the assumption that each production facility produces the same number of DinoBalls each month.

The excel model identified a feasible solution for the base case optimization problem. The model optimizes the cost of manufacturing and transporting DinoBalls to meet customer demand at $19,899.47 during calendar year 2019. Additionally, the model predicts an industrial emissions of 164.81 units during calendar year 2019. The volume of shipment from production facility to retail store varied monthly based on customer demand for that month. Certain shipping routes emerge as frequent choices, while other shipping routes are rarely, or never used. Figure XX below shows the volume of DinoBalls shipped from each production facility to each customer store, each month.

Base Case Shipment table,

Base Case Total Costs

Base Case Total Emissions

# What-If #1 Case Results

## Interpretation

## Constraints, Limitations, Assumptions

## Results

## Sensitivity Analysis

## Major Conclusions

# What If #2 Case Results

## Interpretation

## Constraints, Limitations, Assumptions

## Results

## Sensitivity Analysis

## Major Conclusions

# What If #3 Case Results

## Interpretation

## Constraints, Limitations, Assumptions

## Results

## Sensitivity Analysis

## Major Conclusions

# Conclusions

Take Aways

Answer All Remaining Questions: Optimal Soln, Shadow Price

Future Research

* Will FastProd allow us to reduce our employment workforce. Since there is no hourly charg for production, Fast Pro not imporoving cost but thoggh improving time, we have not quanitified time saved in terms of cost. Also consider, as long as each factory meets one third demand, they are not every really reachin ght peoductio limito f their factor eeven on a multi year demand schedule where you would increase. As long as each factory produces equal demand the production level is feasible.
* They didn’t account for maintenance of the production equipment in terms of cost. You could model this as an LP in the future as a revenue minus cost function, where cost is represented in some sort of time cost unit for each assembly line. We don’t worry about trucks. Variable gas prices.
* If they wanna use fastprod and relax shared demand, if demand increase over years, it MAY save money in long run. Because right now no prod facility even with uneven demand is capping out its production capacity. Only time fast prod is useful in all scenarios is where you exceeed your current production capacities at each facility.
* Did we consider whether or not selling the production lines would result in a rebate and reduce cost if implementing fast prod as opposed to not selling, staying with same deal.

**Appendices**

**Put all data tables in here and link back to report reference (especially the data tables of energy use, production cost, line cost, ship cost, per unit ship cost, truck et al).**

**References**

**Ahmed, M & Khan, Aminur & Uddin, Md & Ahmed, Faruque. (2016). A New Approach to Solve Transportation Problems. Open Journal of Optimization. 05. 10.4236/ojop.2016.51003.**

**Ishaq Abu Halawa, Moh’d & Maatuk, Abdelsalam & Saleh Idrees, Heba & Ali, Eman. (2016). An Optimal Solution for Transportation Problem Using Computing Modelling. 10.1109/ICEMIS.2016.7745340.**

**--------------------**

Methodology and overall approach goes here.

Shell seems fine.

Clarify: uneven, how to handle the non intenger soln of toys: take care of integrality in some place. Where can you enforce integrality.

Argument: not worth to force integrality. Non forcing gives more conservative estimate. Not just deciding whether to get rid but what is result if constraint remains. Factions of dollars, so if that’s w

Right track with danie’s logic. Need a little wiggle room, to get an integer solution.

Inherent; problems forcing every line to produce integer level toys. Cant’ say not worth doing, but real reasons may not be able to do that. F

Wear tear does not become binding if done judged by hours.

Cox: Constraint is that eq wear tear in hour. Explore in report down path of implications of if I need to produce same # toys every line, run same amt of time, what is this forcing down. Why do I maybe need to not force integrality then…If you force integrality, infeasible.

Cox: if you want to do this, then I’m going to relax equal time const. and force integrality. Sensitivity anlys. Can come up with different logical

Exec Summary – if can reasonably figure handful of key results – htat would be ideal. b/c. Exec:wjat should go in quad chart to the CEO.

ASS:

- doesn’t have to be Linear, but specify if not.

-pareto curve cost vs. pollution.

-Cox: reality situation is that politics play role but we are presenting our analysis, numeric driven.

-Trucks don’t require full load. How does that impact soln: is truck going a sunk cost (whether you use full truck or not). E.g. Truck cost, 20 percent, and per unit ship cost.

-Put figures in body.

-Sales price: whatever decision if they want to discount, by switch to fast prod, have to have a 3 year outlook. Cox: three base prod lines, and replace 2/3. Sell the others. Obvious good bad lines. Might obvious answer to replace bad lines, but what if you can get 100x for selling the good line, would jacking up the sale price of a good line and replacing with fast prod offset? Need to address this in some form fashion. How sensitive is the who sell old line and fast prod upgrade tradeoff on the objective.

-Don’t know how much workers under fast prod are getting paid.

-Excursions, trails away from results – put in appendices. Don’t care about code. Visual displays, tables should be embedded in report.

Cox: -Sales price: if you go to fast prod: keep, all break down, how are we going to handle sales price;

Cox does not really care about seeing code but attach as appendix as desired. Perhaps some lingo excel example output of case.

Cox: as long as it is logically defensible you may argue it.

XS: put the dollar figures in. The shipping quantities and tables you can put in results. I would go for a quad chart.graphic to reduce the nonsense of formatting tables in the report.

Cox: Each section, cases are independent. Sensitivity analysis in each section should stand alone. DO not reference gobbledygook in other section.

Not everyone is required to present.

Clear, concisde, explain assertions and reference the charts if you make an assumption. Reference assumptions.

Cox Feedback:

The Sevenworks team then examined the data given on the supply chain network for DinoBall that was provided by MCS. This included a data review of the following information about the supply chain.

|  |  |
| --- | --- |
| * Monthly Demand Figures Each Store | * Shipping Distances, ProdFac to Store |
| * Fuel Cost from ProdFac to Store, Dollars | * Units of Fuel Rqmts, ProdFac to Store |
| * Production Line Specifications | * New Technology FastProd Specifications |
| * + Output, Hourly Energy | * + Output, Hourly Energy |
| * + Hourly Energy Use | * + Hourly Energy Use |
| * + Per Unit Production Cost | * + Per Unit Production Cost |
| * + Per Product Emissions Output | * + Per Product Emissions Output |

Sevenworks input this data into tabular form in order to use as inputs for a model that required inputs of that information. Before approaching model formulation alternative, Sevenworks derived from this data the **constraints** of the supply chain problem:

|  |  |
| --- | --- |
| * Max Supply, Output from Production Facility, Monthly (The Production Limit) | * Necessity of shipping whole toys (no shipping of fractional toys) |
| * Trucks Ship a Max of 250 Units per Trip   + ProdFac to Store in a given month | * Each production facility must share total demand. |

We considered from the data and MCS corporate guidance the **limitations** of modeling the supply chain for DinoBall to include these possible limitations in modeling:

|  |
| --- |
| * The one third demand constraint my cause the shipment of fractional, non-integer DinoBall units. |
| * Identified we need to address the issue of integrality because of the requirements of toys being fractional and given we do not usually ship fractions of products with determining solution. The residual between the integer and non-integer decision value for the decision variable in solution. |
| * Placeholder for additional limitations. |

This means that when implementing a mathematical solution, given the constraint of production equal between each production facility, in combination with whole or integer units of demand at each store, that a solution may not be an integer. In other words, the model ships fractional toys. While not ideal, the Sevenworks concluded it may be necessary in the model to output a solution with fractional, non-integer, shipments before post analysis.

In order to the scope the problem and ensure modeling feasibility in a timely manner, Sevenworks, converged on several assumptions.

|  |
| --- |
| * The provided demand, fuel cost/requirements, production line specification, distances, are constant, and reliable |
| * We assume equal wear and tear on production lines irregardless of quantity produced |
| * We assume no transit delays between the production facilities and the stores via delivery trucks. |
| * Placeholder for additional assumptions (this is a linear program should go here? It is in base case..? |